Plug-and-Play Distributed Control of Large-Scale Nonlinear Systems

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***Abstract*—A method to design plug-and-play (PnP) distributed controllers for large-scale nonlinear systems represented by interconnected Takagi–Sugeno fuzzy models with nonlinear consequent is presented in this article. From the combination of techniques to use multiple fuzzy summations and to explore the chordal decomposition of the interconnection graph associated with the large-scale nonlinear system, sufficient conditions for distributed stabilization are derived in terms of linear matrix inequalities (LMIs). Conditions specially designed to allow seamless subsystems plugging-in and unplugging operations from the large-scale system, without requiring the redesign of all previously tuned distributed controllers, are provided. The approach can be used together with fault detection and isolation (FDI) systems, and also in the context of mixed distributed and decentralized controllers operating in a network of interconnected systems. To illustrate the effectiveness of the proposed PnP approach, a network of nonlinearly coupled and heterogeneous Van der Pol oscillators is used in the numerical experiments.**

***Index Terms*—Chordal decomposition, distributed control, large-scale systems (LSSs), plug-and-play (PnP) control, Takagi–Sugeno (TS) fuzzy systems.**

I. Introduction

LARGE-SCALE systems (LSSs) have attracted increasing attention from researchers in the last decades, either because of their extensive applications in various engineering systems, such as communication networks, power systems, mobile robots, industrial processes, transportation networks, or due to the increase in the complexity of society supporting systems, thanks to the growing environmental challenges and recent technological solutions based on advances in communication, control, and automation [1], [2].

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In general, LSSs consist of a large number of interconnected subsystems, which interact with each other and are spatially distributed. Frequently, the problem of LSSs stability analysis becomes very challenging because of information structure constraints, uncertainty, and induced delays [1]. Although there is no precise notion of large-scale, we consider a system to be large scale when its dimensions are so large that conventional techniques of modeling, analysis, control, and computation fail to provide solutions with reasonable computational effort such that it becomes necessary to use some decomposition techniques to address the problem of system stability analysis [3].

Particularly, the problem of taking into account information on the constraints in the underlying interconnection structure is related to the task of defining appropriate control strategies consistent with the information flow between subsystems. In this context, decentralized control structures have been proposed in different scenarios as an alternative to this problem (see surveys [4], [5] and references therein).

However, when the interconnections among subsystems are very strong and the local controllers are designed not taking this into account, the decentralized control may not be effective to ensure suitable levels of performance for LSSs and, sometimes, proper subsets of local controllers that can stabilize the overall network might not even exist [1]. Distributed control schemes arise to overcome these issues, where local controllers can make use of information on internal variables along with those of the subsystems in the neighborhood of the locally controlled subsystem to compute its control input, providing greater reliability with respect to communication failures in the network in comparison to centralized controllers, while still being able to improve the overall system performance and stability in comparison to decentralized controllers [6]. Recently, distributed schemes have been proposed in different applications, such as distributed event-triggering control [7], [8]; distributed states estimation network [9], [10]; and consensus of multiagent systems [11].

The chordal decomposition [12] has been used to reduce the complexity of sparse semidefinite programs as in [13], where it is guaranteed near-linear time complexity for off-the-shelf interior-point methods implemented in SeDuMi and MOSEK. However, for the controller design, only linear systems have been considered to specific applications, such as distributed robustness analysis of interconnected uncertain systems [14]; design of distributed control of interconnected linear systems [15], [16]; and decentralized control design to weakly coupled linear systems [17]. Thus, to the best of our knowledge, there exists a lack in the literature on the possible advantages of using chordal decomposition to obtain distributed controllers for LSSs composed of nonlinear subsystems.

Another significant challenge in designing controllers for LSSs is dealing with the presence of nonlinear interconnections between subsystems. In general, only linear interconnections are considered in many works in the current literature on this subject, or they are even sometimes considered as exogenous disturbances and altogether neglected in the control design task. On the other hand, when nonlinear interconnections are taken into consideration, they usually have to satisfy a few particular conditions. For instance, a Lipschitz condition was considered in [9] and [18] and polynomial bounding conditions were assumed in [19] and [20]. Uncertain norm-bounded interconnections were considered in [21] and [22], while a quadratic bounding was used in [23], in the context of dynamic and static output feedback, respectively. In [24], radial basis function neural networks were employed to approximate the interconnection functions.

Some of the previous papers mentioned in the last paragraph consider the Takagi–Sugeno (TS) fuzzy framework [25], [26], which allows the use of the Lyapunov stability theory and linear matrix inequalities (LMIs) to obtain sufficient conditions for stability analysis and control synthesis for nonlinear systems. In this case, only the local nonlinearity of subsystems are represented by fuzzy rules, while the nonlinear interconnections satisfy the aforementioned particular conditions. Indeed, if each nonlinear interconnection is transformed into a set of fuzzy rules, this can lead to the so-called rule-explosion problem [27]. To avoid this problem, TS fuzzy models with nonlinear consequent (N-TS fuzzy models) can be used to represent interconnected systems when their interconnections are sector-bounded nonlinear functions [28]. In a similar vein, Vu and Wang [29], [30] also used the TS fuzzy model with polynomial consequent and sum-of-squares (SOS)-based techniques.

Furthermore, LSSs can often have their structure changed along time. Specific characteristics are desirable, as the ability to *add* and *remove* subsystems without the requirement of shutting down the entire system (or part of it) for controllers reparametrization. These ideas are called plug-and-play (PnP) methods in the literature (for more details, see [31] and references therein).

The PnP approach was initially introduced for fault detection and isolation (FDI), as in [32] and [33] that focus on distributed FDI methods for discrete-time LSS. In the context of the control design, in the last few years, some PnP control strategies were published in [34] and [35], in which the design of PnP decentralized model predictive control (MPC) for discrete-time linear systems is investigated, under the assumption of sufficiently weak couplings among subsystems, that is, interconnections are considered as disturbances. Decentralized control for linear subsystems modeled in the frequency domain was proposed in [36]. Yang *et al.* [37] proposed a passive fault-tolerant control scheme for nonlinear continuous-time interconnected systems with interconnections satisfying a Lipschitz condition. In [38], fault detection and distributed MPC for nonlinear LSSs are addressed. The proposed controller is based on the feedback linearization method, where nonlinearities are canceled in the state equations. More recently, the PnP control design for discrete-time linear systems was proposed in [39] and [40], where the decentralized controller and distributed MPC based on the dissipativity approach are considered, respectively.

Note that most of the papers address discrete-time linear systems. Furthermore, those that deal with nonlinear systems do not use TS fuzzy models to represent them. This is due to the combination of membership functions of the subsystems, which makes impossible to obtain a set of finite LMIs conditions numerically tractable when the number of subsystems is large. Based on the previously discussed issues, this article proposes a PnP distributed control strategy for the stabilization of continuous-time large-scale nonlinear systems represented by interconnected TS fuzzy models with nonlinear consequents. Thus, the main contributions of this article are summarized as follows.

1) Using the combination of techniques associated with multiple fuzzy summations, chordal decomposition, and block-diagonal Lyapunov functions, we extend the design of distributed controllers for continuous-time large-scale nonlinear systems described by interconnected N-TS fuzzy systems in [28] to the case where the number of subsystems is large, such that the rule-explosion problem can be avoided.
2) We take advantage of the properties of chordal graphs to propose a new PnP distributed control approach for continuous-time large-scale nonlinear systems. In contrast to methods used in [34] and [35], we deal with continuous-time nonlinear systems with nonlinear interconnections.
3) The presented PnP procedure is flexible and can also be used in the design of mixed distributed and decentralized controllers. Also, it can potentially be used in conjunction with any underlying FDI strategy, which makes it novel with respect to the approaches in [32] and [33].

The remainder of this article is organized as follows. The concepts of chordal graphs and interconnected N-TS fuzzy models are introduced in Section II. In Section III, the distributed control problem is presented. In Section IV, new stabilization conditions for LSSs are presented, while in Section V, we propose a PnP operation procedure. Simulation results are presented in Section VI. Finally, Section VII draws the conclusion.

Notation: The symbol "$\star$" denotes matrix blocks deduced by symmetry. I denotes an identity matrix of appropriate dimension, while $\mathbf{1}_{m\times n}$ and $\mathbf{0}_{m\times n}$ denote $m\times n$ all ones and all zeros matrices, respectively. For matrix X, $X^\top$ is its transpose matrix and $X \succ 0$ means that $X \in \mathbb{S}^n$ is an $n\times n$ symmetric positive-definite matrix. $\text{diag}(X_1, X_2)$ denotes a block-diagonal matrix composed by X_1 and X_2. $\boldsymbol{X} = \{X_{ij}\}$ stands a block matrix. $\boldsymbol{X}[\alpha, \beta]$ is a submatrix of $\boldsymbol{X}$, obtained by choosing the blocks of $\boldsymbol{X}$, which lie in rows α and columns β; α and β are index sets of the rows and the columns of $\boldsymbol{X}$, respectively. Symbol $\circ$ denotes the elementwise or Hadamard product of matrices. The $Y^{\circ(-1)}$ operator denotes the elementwise inverse operation on the Y matrix elements. For an integer $n > 1$, we denote $\mathcal{I}_n = \{1, \ldots, n\} \subset \mathbb{N}$. Function arguments are omitted when their meaning is straightforward. Notation related to chordal graphs is based on [12] and [16].

II. Preliminaries

A. Chordal Graphs

A graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is composed by a set of vertices $\mathcal{V} = \{1, 2, \ldots, n\}$ and a set of edges $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$, with each edge representing a connection from vertex i to vertex j. A graph is *undirected* if it represents mutual interactions between vertices, that is, $(i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$.

A path is a sequence of edges that connects a sequence of distinct vertices. A graph is called *connected* if there is a path between every pair of vertices. A *clique* is a subset of vertices $\mathcal{C} \subseteq \mathcal{V}$ that induces a *complete* subgraph $\mathcal{G}_{\mathcal{C}}(\mathcal{C}, \mathcal{E}_{\mathcal{C}})$, that is, $(i, j) \in \mathcal{E}_{\mathcal{C}}$ for any distinct vertices $i, j \in \mathcal{C}$. If $\mathcal{C}$ is not a subset of any other clique, then it is called a *maximal clique*. The number of vertices in $\mathcal{C}$ is denoted by $|\mathcal{C}|$. A *cycle* of length k in a graph $\mathcal{G}$ is a set of pairwise distinct vertices $\{1, 2, \ldots, k\} \subseteq \mathcal{V}$ such that $(k, 1) \in \mathcal{E}$ and $(i, i+1) \in \mathcal{E}$ for $i = \{1, 2, \ldots, k-1\}$. A *chord* is an edge between nonconsecutive vertices on a path. In a cycle, a chord is an edge connecting two nonadjacent vertices.

Definition 1 (Chordal Graph [12]): An undirected graph $\mathcal{G}$ is called *chordal* if every cycle of length greater than 3 has at least one chord.

Nonchordal graphs can always be *extended* to a chordal graph by adding new edges to the original one. A chordal extension can be efficiently generated from heuristics, such as the minimum degree ordering followed by a symbolic Cholesky factorization [12].

Definition 2 (Chordal Extension [12]): A *chordal extension* of a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a chordal graph $\hat{\mathcal{G}}(\mathcal{V}, \hat{\mathcal{E}})$, where $\mathcal{E} \subseteq \hat{\mathcal{E}}$.

Given an undirected graph $\mathcal{G}$, its corresponding adjacency matrix $A_{\mathcal{G}} = \{a_{ij}\} \in \mathbb{R}^{n \times n}$ is such that $a_{ij} = 1$, if $(i, j) \in \mathcal{E}$, indicating a connection (edge) between vertices i and j, and $a_{ij} = 0$, otherwise. Since the graph is undirected, the adjacency matrix is symmetric. In addition, the neighborhood of the ith vertex is defined by the set of vertices $\mathcal{N}_i = \{j \in \mathcal{V} : a_{ij} \neq 0\}$. The degree matrix of a graph $D_{\mathcal{G}} = \{d_{ij}\} \in \mathbb{R}^{n \times n}$ is a diagonal matrix that contains information on the number of edges attached to each vertex, such that $d_{ii} = \sum_{j=1}^{n} a_{ij}\ \forall i \in \mathcal{V}$; and $d_{ij} = 0$, if $i \neq j$.

A vertex of an undirected graph is called *simplicial* if the subgraph induced by its neighborhood $\mathcal{N}_i$ is complete, that is, all its neighbors are connected to each other. An ordering (or equivalently a numbering of the vertices) $\sigma = \langle 1, \ldots, n \rangle$ of an undirected graph $\mathcal{G}$ is a *perfect elimination ordering* if each ith vertex, for $i = \{1, 2, \ldots, n\}$, is a simplicial vertex in the subgraph induced by the vertices $\{i, i+1, \ldots, n\}$. Let $\mathcal{G}$ be a connected chordal graph. Given its perfect elimination ordering, the set of maximal cliques $\Sigma = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$ of the graph can be identified in linear time [12].

B. Sparse Block Matrices and Chordal Decomposition

The occurrence of block matrices is almost inevitable when investigating LSSs. Therefore, it is necessary an extension of the Chordal Decomposition Theorem [41] for the case where X is a block matrix, that is, $\boldsymbol{X} = \{X_{ij}\}$. This extension was presented in [15]. However, we will define a more general class of block matrices, which is necessary for the developments in Sections IV and V.

Given the sets of vectors $\boldsymbol{\delta} = \{\delta_1, \ldots, \delta_N\}$ and $\boldsymbol{\lambda} = \{\lambda_1, \ldots, \lambda_N\}$, with $\delta_i, \lambda_i \in \mathbb{R}$, $i \in \mathcal{I}_N$, a block matrix $\boldsymbol{X} \in \mathbb{R}^{n_\phi \times n_x}$ has $(\boldsymbol{\delta\lambda})$-partitioning with $n_\phi = \sum_{i=1}^{N} \delta_i$ and $n_x = \sum_{i=1}^{N} \lambda_i$, if each block $X_{ij} \in \mathbb{R}^{\delta_i \times \lambda_j}\ \forall i, j \in \mathcal{I}_N$. The space of $(\boldsymbol{\delta\lambda})$-partitioned matrices $\boldsymbol{X}$ with sparsity pattern given by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is

$$\mathbb{R}^{n_\phi \times n_x}_{\boldsymbol{\delta\lambda}}(\mathcal{E}, 0) = \{\boldsymbol{X} \in \mathbb{R}^{n_\phi \times n_x} : X_{ij} = 0,\ \text{if } (i, j) \notin \bar{\mathcal{E}}\}$$

with $\bar{\mathcal{E}} = \mathcal{E} \cup \{(i, i)\ \forall i \in \mathcal{V}\}$. For the case $\boldsymbol{\delta} = \boldsymbol{\lambda}$, the previous space is reduced to the one presented in [15], in the sense that $n_\phi = n_x$. For an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, the space of $\boldsymbol{\lambda}$-partitioned symmetric matrices with sparsity pattern $\mathcal{E}$ is defined as

$$\mathbb{S}^{n_x}_{\boldsymbol{\lambda}}(\mathcal{E}, 0) = \left\{\boldsymbol{X} \in \mathbb{S}^{n_x} : X_{ij} = 0,\ \text{if } (i, j) \notin \bar{\mathcal{E}}\right\}.$$

If $\lambda_i = 1\ \forall i \in \mathcal{I}_N$, the space defined above will be denoted by $\mathbb{S}^{n_x}(\mathcal{E}, 0)$.

The following lemma extends the chordal decomposition theorem to the case of block matrices.

Lemma 1 (Block-Chordal Decomposition Theorem [17]): Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a chordal graph with t maximal cliques $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$. Given a partition $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ and $n_x = \sum_{i=1}^{N} \lambda_i$, then, $\boldsymbol{X} \in \mathbb{S}^{n_x}_{\boldsymbol{\lambda}}(\mathcal{E}, 0)$ is a positive semidefinite matrix if and only if there exist positive semidefinite matrices $\boldsymbol{X}^k \in \mathbb{S}^{|\mathcal{C}_k|_{\boldsymbol{\lambda}}}$, with $|\mathcal{C}_k|_{\boldsymbol{\lambda}} = \sum_{i \in \mathcal{C}_k} \lambda_i$, for $k \in \mathcal{I}_t$ such that

$$\boldsymbol{X} = \sum_{k=1}^{t} (\boldsymbol{E}_{\mathcal{C}_k})^\top \boldsymbol{X}^k \boldsymbol{E}_{\mathcal{C}_k}$$

where $\boldsymbol{E}_{\mathcal{C}_k} = \boldsymbol{E}[\mathcal{C}_k, \mathcal{V}] \in \mathbb{R}^{|\mathcal{C}_k|_{\boldsymbol{\lambda}} \times n_x}\ \forall k \in \mathcal{I}_t$, are blockwise principal submatrices of $\boldsymbol{E} = \operatorname{diag}(I_{\lambda_1}, \ldots, I_{\lambda_N})$.

Remark 1: The chordal decomposition theorem plays an important role in the context of sparse semidefinite problems since if an LMI constraint has a chordal sparsity pattern, then it can be equivalently replaced by a set of LMI constraints using matrices with smaller dimensions, together with a set of equality constraints. Therefore, according to [13], [42], and [43], this theorem brings substantial computational enhancement to solving large sparse semidefinite problems, especially if the numbers of vertices in maximal cliques are small.

Remark 2: Note that the block-chordal decomposition theorem is only applied when the graph is chordal, and if it is nonchordal its chordal extension $\hat{\mathcal{G}}(\mathcal{V}, \hat{\mathcal{E}})$ (see Definition 2) is used instead to find the set of maximal cliques in Lemma 1. In addition, the application of Lemma 1 generates a set of equality constraints associated with overlapping elements in the graph, that is, blocks in $\boldsymbol{X}^k$ that correspond to a vertex can appear in more than one constraint when this vertex belongs to more than one maximal clique.

To eliminate equality constraints, we define matrices that equally divide the respective repeated block between the constraints to which it belongs. For this purpose, we define $Z = \{z_{ij}\} \in \mathbb{S}^N(\hat{\mathcal{E}}, 0)$, considering the set of maximal cliques of the graph $\Sigma = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$, such that

$$\begin{cases} z_{ii} = \text{ number of repetitions of vertex } i \text{ in } \Sigma \\ z_{ij} = \text{ number of repetitions of edge } (i,j) \text{ in } \Sigma. \end{cases} \quad (1)$$

Thus, a matrix of averaging factors for decomposing the overlapping elements can be defined as follows:

$$\boldsymbol{V} = \{V_{ij}\}, \quad \text{with } V_{ij} = \begin{cases} \frac{1}{z_{ij}} \times \mathbf{1}_{\lambda_i \times \lambda_j}, & \text{if } z_{ij} \neq 0 \\ \mathbf{0}_{\lambda_i \times \lambda_j}, & \text{otherwise} \end{cases}$$

such that $\boldsymbol{X}^k = \boldsymbol{V}[\mathcal{C}_k, \mathcal{C}_k] \circ \boldsymbol{X}[\mathcal{C}_k, \mathcal{C}_k]$ in Lemma 1.

Note that when $\lambda_i = 1\ \forall i \in \mathcal{I}_N$, Lemma 1 is reduced to the traditional chordal decomposition theorem [41]. In this article, to each element in a given partition corresponds the order of the associated subsystem, that is, $\lambda_i = n_{x_i}$.

C. Interconnected N-TS Fuzzy Models

When classical TS fuzzy models [25] are used to represent complex dynamical systems, an explosion in the number of fuzzy rules may occur due to the possible presence of a large number of nonlinearities. Stability analysis and control synthesis for such TS fuzzy models are often very challenging because of the high computational complexity usually associated with a large number of rules. Although the fuzzy local approximation method can be employed to obtain nonexact TS fuzzy models with fewer fuzzy rules, the designed control laws may not guarantee the stability of the original nonlinear system [27].

Therefore, N-TS models, that is, TS fuzzy models with nonlinear consequent, have been employed to avoid using an excessive number of fuzzy rules while increasing the model accuracy. In this sense, Dong *et al.* [44] proposed adding sector-bounded functions to the traditional TS fuzzy models, resulting in new models with nonlinear consequent. By using this approach, the following continuous-time and input affine nonlinear system:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t)) + g(\mathbf{x}(t))\mathbf{u}(t)$$

where $f(\cdot)$ and $g(\cdot)$ are smooth nonlinear functions, such that $\mathbf{x}(t) = 0$ and $\mathbf{u}(t) = 0$ are the equilibrium conditions for the nonlinear system, which can be represented as an N-TS fuzzy model

$$\dot{\mathbf{x}}(t) = \sum_{k=1}^{r} \varsigma_k(\mathbf{z}(t))\big[A_k\mathbf{x}(t) + B_k\mathbf{u}(t) + G_k\boldsymbol{\varphi}(\mathbf{x}(t))\big] \quad (2)$$

where r is the number of fuzzy rules, $\mathbf{z}(t) \in \mathbb{R}^p$ is the premise variables vector, and $\varsigma_k(\mathbf{z}(t))$ the membership functions, $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state vector, $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the control input vector, $\boldsymbol{\varphi}(\mathbf{x}(t)) \in \mathbb{R}^{n_\varphi}$ is a vector of sector-bounded nonlinear functions, and $A_k \in \mathbb{R}^{n_x \times n_x}$, $B_k \in \mathbb{R}^{n_x \times n_u}$, and $G_k \in \mathbb{R}^{n_x \times n_\varphi}$ are constant matrices describing the local dynamics of the system.

As a result, both conservativeness and computational complexity for stability analysis and control design can be reduced when using N-TS fuzzy models for continuous-time [44], [45] and discrete-time systems [46], [47]. N-TS fuzzy models were used for the first time to represent interconnected nonlinear systems in [28], with the sector-bounded nonlinear functions representing interconnections among subsystems in a network.

Following this approach, consider that each ith subsystem in a continuous-time large-scale nonlinear system with N heterogeneous interconnected nonlinear subsystems, whose nonlinear interconnections are associated with an undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ can be described as follows:

$$\dot{\mathbf{x}}_i(t) = f_i(\mathbf{x}_i(t)) + g_i(\mathbf{x}_i(t))\mathbf{u}_i(t) + \sum_{j \in \mathcal{N}_i} h_{ij}\big(\mathbf{x}_i(t), \mathbf{x}_j(t)\big)$$

where $i \in \mathcal{I}_N$, $\mathbf{x}_i(t) \in \mathbb{R}^{n_{x_i}}$ is the state vector for the ith subsystem, and $\mathbf{u}_i(t) \in \mathbb{R}^{n_{u_i}}$ is the ith control input vector. An interconnected N-TS fuzzy model can then be obtained using the same strategy in [44], by considering that nonlinear interconnections among subsystems are represented by nonlinearities $\boldsymbol{\varphi}$ in (2), such that the ith inferred N-TS fuzzy model is given by

$$\dot{\mathbf{x}}_i(t) = \sum_{l=1}^{r_i} \varsigma_i^l(\mathbf{z}_i(t))\Bigg[A_{ii}^l\mathbf{x}_i(t) + B_i^l\mathbf{u}_i(t) + \sum_{j \in \mathcal{N}_i} A_{ij}^l\mathbf{x}_j(t) + G_i^l \sum_{j \in \mathcal{N}_i} \varphi_{ij}\big(\mathbf{x}_i(t), \mathbf{x}_j(t)\big)\Bigg] \quad (3)$$

where $i \in \mathcal{I}_N$ and $l \in \mathcal{I}_{r_i}$, with r_i the number of rules, $\mathbf{z}_i(t) \in \mathbb{R}^{p_i}$ is the premise variables vector associated to the ith subsystem, $\varphi_{ij}(\cdot) \in \mathbb{R}^{n_{\varphi_{ij}}}$ are sector-bounded functions describing the nonlinear interconnections between the ith and jth subsystems, and A_{ii}^l, B_i^l, A_{ij}^l, and G_i^l are known constant local matrices with appropriate dimensions.

Remark 3: Note that in the general framework being presented so far, each function φ_{ij} in (3) may have a different dimension $n_{\varphi_{ij}}$, which would lead to difficulties in writing the equations compactly. Thus, from now on, without loss of generality and to maintain a concise development, we will consider $\varphi_{ij} \in \mathbb{R}$.

By considering real-valued nonlinear functions φ_{ij}, we can define decentralized vectors of nonlinearities

$$\boldsymbol{\phi}_i(\mathbf{x}(t)) = \begin{bmatrix} \varphi_{ij_1} & \varphi_{ij_2} & \cdots & \varphi_{ij_{d_{ii}}} \end{bmatrix}^\top \quad (4)$$

such that each $\boldsymbol{\phi}_i(\mathbf{x}(t)) \in \mathbb{R}^{d_{ii}}$ is formed by stacking the nonlinear functions φ_{ij_κ}, with $j_\kappa \in \mathcal{N}_i$, $\kappa \in \mathcal{I}_{d_{ii}}$. This also prevents possible numerical problems in the design conditions, since if the ith and jth subsystems are not interconnected, the nonlinearity φ_{ij} does not exist and it should not be taken into consideration. In addition, since each function φ_{ij} is sector bounded, each decentralized vector of nonlinearities $\boldsymbol{\phi}_i(\mathbf{x})$ also satisfies a corresponding sector property, that is, there exists a given matrix $\boldsymbol{\Omega}_i = \begin{bmatrix} \Omega_{i1}\ \Omega_{i2} \cdots \Omega_{iN} \end{bmatrix} \in \mathbb{R}^{d_{ii} \times n_x}$, $n_x = \sum_{i=1}^N n_{x_i}$, with $\Omega_{ij} \in \mathbb{R}^{d_{ii} \times n_{x_j}}\ \forall j \in \mathcal{I}_N$, and $\Omega_{ij} = 0\ \forall j \notin \mathcal{N}_i$, such that the following inequality holds:

$$\boldsymbol{\phi}_i(\mathbf{x})^\top \Lambda_i^{-1}\big(\boldsymbol{\phi}_i(\mathbf{x}) - \boldsymbol{\Omega}_i\mathbf{x}\big) \leq 0 \quad (5)$$

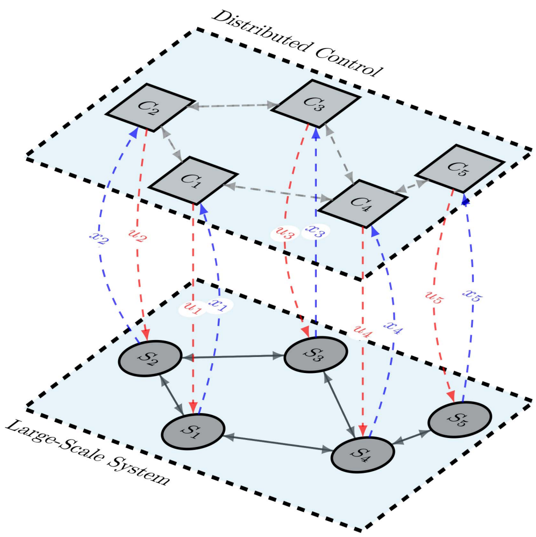

Fig. 1. Distributed control structure. Dashed arrows (gray) between local controllers indicate that the states or control inputs of subsystems in the neighborhood are sent to a local controller to compute the control signal for its corresponding subsystem.

where $\Lambda_i \in \mathbb{R}^{d_{ii}\times d_{ii}}$ is any positive-definite diagonal matrix, which is used here to add degrees of freedom to the above inequality.

III. Distributed Control Problem

It is possible to explore the flexibility in the control design provided by N-TS fuzzy models together with the advantages of a distributed control approach in the design of a nonlinear control law for the LSS represented by (3). This control strategy is distributed in the sense that local controllers can make use of data from their own local subsystem internal variables and from their neighboring subsystems in the computation of their control actions, as shown in Fig. 1. Particularly, we consider that only one undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ represents the interconnections either among subsystems or among local controllers in the control layer.

Consider the following distributed control law for the ith subsystem:

$$\mathbf{u}_i(t) = \sum_{l=1}^{r_i} \varsigma_i^l(\mathbf{z}_i(t))K_i^l\mathbf{x}_i(t) + \sum_{j\in\mathcal{N}_i} F_{ij}\mathbf{x}_j(t) + \Gamma_i\boldsymbol{\phi}_i(\mathbf{x}(t)) \quad (6)$$

where $K_i^l \in \mathbb{R}^{n_{u_i}\times n_{x_i}}$, $F_{ij} \in \mathbb{R}^{n_{u_i}\times n_{x_j}}$, $\Gamma_i \in \mathbb{R}^{n_{u_i}\times d_{ii}}$, that is, $\Gamma_i = \begin{bmatrix}\Gamma_{ij_1} & \Gamma_{ij_2} & \cdots & \Gamma_{ij_{d_{ii}}}\end{bmatrix}$, with $j_\kappa \in \mathcal{N}_i$, $\kappa \in \mathcal{I}_{d_{ii}}$, following the previous definition of $\boldsymbol{\phi}_i(\mathbf{x}(t))$ in (4).

The following assumption is considered in order to have a consistent control law in (6).

Assumption 1: The state vector $\mathbf{x}_j(t)$ is available for the ith subsystem as well as the nonlinear functions $\varphi_{ij}(\mathbf{x}_i, \mathbf{x}_j)$ are known for the ith subsystem, if $j \in \mathcal{N}_i$.

Remark 4: Similar to [44], the nonlinearities $\phi_i(\mathbf{x}(t))$ were incorporated in the nonlinear distributed control law (6). This *a priori* additional information aims to improve the system response by reducing conservativeness in the controller synthesis procedure. The same approach has been used in several works on N-TS fuzzy systems either for continuous-time [44], [45] or for discrete-time systems [46], [47].

Then, substituting (6) in (3) leads to the following closed-loop subsystem:

$$\begin{aligned}\dot{\mathbf{x}}_i(t) = &\sum_{k=1}^{r_i}\sum_{l=1}^{r_i} \varsigma_i^k(\mathbf{z}_i(t))\varsigma_j^l(\mathbf{z}_i(t))\left(A_{ii}^k + B_i^k K_i^l\right)\mathbf{x}_i(t)\\ &+ \sum_{j\in\mathcal{N}_i}\sum_{k=1}^{r_i} \varsigma_i^k(\mathbf{z}_i(t))\left(A_{ij}^k + B_i^k F_{ij}\right)\mathbf{x}_j(t)\\ &+ \sum_{k=1}^{r_i} \varsigma_i^k(\mathbf{z}_i(t))\left(G_i^k \mathbf{1}_{d_{ii}}^\top + B_i^k\Gamma_i\right)\boldsymbol{\phi}_i(\mathbf{x}(t)). \end{aligned} \quad (7)$$

It is known that the Lyapunov inequalities for linear systems admit block-diagonal solutions [48]. Then, we will use a quadratic block-diagonal Lyapunov function candidate for stability analysis of the N interconnected closed-loop subsystems in (7)

$$V(\mathbf{x}(t)) = \sum_{i=1}^{N} \mathbf{x}_i(t)^\top P_i \mathbf{x}_i(t) = \mathbf{x}(t)^\top \mathbf{P}_N \mathbf{x}(t) \quad (8)$$

where $\mathbf{x}(t) = \begin{bmatrix}\mathbf{x}_1(t)^\top & \mathbf{x}_2(t)^\top & \cdots & \mathbf{x}_N(t)^\top\end{bmatrix}^\top$ and $\mathbf{P}_N = \mathrm{diag}(P_1,\ldots,P_N)$, with $P_i = P_i^\top \succ 0\ \forall i \in \mathcal{I}_N$.

Based on the previous discussions, this article proposes solutions to the following problems.

Problem 1: The determination of sufficient conditions for designing distributed nonlinear controllers (6) for the stabilization of closed-loop subsystems (7) that are the constituents of a large-scale nonlinear system.

Problem 2: The determination of sufficient conditions for the distributed stabilization of large-scale nonlinear systems from a *PnP approach*, that is, considering that subsystems can be seamless added or removed from the network.

IV. Distributed Stabilization of Large-Scale Systems

The next theorem provides sufficient conditions to ensure that the origin of the large-scale nonlinear system is asymptotically stable. As the theorem addresses all subsystems of the entire network, the notation of multi-index presented in the Appendix will be used.

Theorem 1: Let the matrices $\mathbf{\Omega}_i$ in (5) be given $\forall i \in \mathcal{I}_N$. The continuous-time large-scale nonlinear system represented by the undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ and formed by N interconnected closed-loop subsystems given in (7) is stable if there exist matrices $Q_i \succ 0$, diagonal matrices $\Lambda_i \succ 0$, and any matrices $R_i^{\iota_{i,2}}$, S_{ij}, and T_i $\forall i \in \mathcal{I}_N$ $\forall \iota_{i,2} \in \mathcal{I}_{r_i}$, and $\forall j \in \mathcal{N}_i$, satisfying the following constraint:

$$\prod_{i=1}^{N}\left(\sum_{\iota_{i,1}=1}^{r_i}\sum_{\iota_{i,2}=1}^{r_i} \varsigma_i^{\iota_{i,1}}\varsigma_i^{\iota_{i,2}}\right)\begin{bmatrix}\mathbf{\Theta} & \star\\ \mathbf{\Pi} & -2\mathbf{\Lambda}_N\end{bmatrix} \prec 0 \quad (9)$$

where $\mathbf{\Lambda}_N = \mathrm{diag}(\Lambda_1,\ldots,\Lambda_N)$, $\mathbf{\Theta} = [\Theta_{ij}] \in \mathbb{S}_{\boldsymbol{\lambda}}^{n_x}(\mathcal{E},0)$, and $\mathbf{\Pi} = [\Pi_{ij}] \in \mathbb{R}_{\boldsymbol{\delta\lambda}}^{n_\phi\times n_x}(\mathcal{E},0)$, with $\boldsymbol{\delta} = \{d_{11},\ldots,d_{NN}\}$, $\boldsymbol{\lambda} =$

$\{n_{x_i}, \ldots, n_{x_N}\}$, and

$$\Theta_{ii} = A_{ii}^{\iota_{i,1}} Q_i + B_i^{\iota_{i,1}} R_i^{\iota_{i,2}} + (A_{ii}^{\iota_{i,1}} Q_i + B_i^{\iota_{i,1}} R_i^{\iota_{i,2}})^\top$$
$$\Theta_{ij} = A_{ij}^{\iota_{i,1}} Q_j + Q_i (A_{ji}^{\iota_{j,1}})^\top + B_i^{\iota_{i,1}} S_{ij} + S_{ji}^\top (B_j^{\iota_{j,1}})^\top$$
$$\Pi_{ii} = \left(B_i^{\iota_{i,1}} T_i + G_i^{\iota_{i,1}} \mathbf{1}_{d_{ii}}^\top \Lambda_i\right)^\top + \Omega_{ii} Q_i$$
$$\Pi_{ij} = \Omega_{ij} Q_j.$$

Then, the control gains in (6) are recovered from:

$$K_i^{\iota_{i,1}} = R_i^{\iota_{i,1}} Q_i^{-1}, \;\; \Gamma_i = T_i \Lambda_i^{-1} \text{ and } F_{ij} = S_{ij} Q_j^{-1}.$$

Proof: If inequality (9) holds, taking $R_i^{\iota_{i,1}} = K_i^{\iota_{i,1}} Q_i$, $T_i = \Gamma_i \Lambda_i$, and $S_{ij} = F_{ij} Q_j$ $\forall i \in \mathcal{I}_N$ $\forall \iota_{i,1} \in \mathcal{I}_{r_i}$, and $\forall j \in \mathcal{N}_i$; and applying the congruence transformation $\text{diag}(\boldsymbol{P}_N, \boldsymbol{\Lambda}_N^{-1})$, leads to the inequality:

$$\prod_{i=1}^{N} \left(\sum_{\iota_{i,1}=1}^{r_i} \sum_{\iota_{i,2}=1}^{r_i} \varsigma_i^{\iota_{i,1}} \varsigma_i^{\iota_{i,2}} \right) \begin{bmatrix} \bar{\boldsymbol{\Theta}} & \star \\ \bar{\boldsymbol{\Pi}} & -2\boldsymbol{\Lambda}_N^{-1} \end{bmatrix} \prec 0 \qquad (10)$$

with

$$\bar{\Theta}_{ii} = P_i\big(A_{ii}^{\iota_{i,1}} + B_i^{\iota_{i,1}} K_i^{\iota_{i,2}}\big) + \big(A_{ii}^{\iota_{i,1}} + B_i^{\iota_{i,1}} K_i^{\iota_{i,2}}\big)^\top P_i$$
$$\bar{\Theta}_{ij} = P_i A_{ij}^{\iota_{i,1}} + \left(A_{ji}^{\iota_{i,2}}\right)^\top P_j + P_i B_i^{\iota_{i,1}} F_{ij} + F_{ji}^\top \left(B_j^{\iota_{i,2}}\right)^\top P_j$$
$$\bar{\Pi}_{ii} = \left(B_i^{\iota_{i,1}} \Gamma_i + G_i^{\iota_{i,1}} \mathbf{1}_{d_{ii}}^\top\right)^\top P_i + \Lambda_i^{-1} \Omega_{ii}$$
$$\bar{\Pi}_{ij} = \Lambda_i^{-1} \Omega_{ij}.$$

Premultiplying and postmultiplying (10) by $\begin{bmatrix} \mathbf{x}^\top & \boldsymbol{\phi}(\mathbf{x})^\top \end{bmatrix}$ and its transpose, and considering (7), one has that

$$2 \sum_{i \in \mathcal{I}_N} \dot{\mathbf{x}}_i(t)^\top P_i \mathbf{x}_i(t) - 2 \sum_{i \in \mathcal{I}_N} \boldsymbol{\phi}_i(\mathbf{x})^\top \Lambda_i^{-1} \big(\boldsymbol{\phi}_i(\mathbf{x}) - \boldsymbol{\Omega}_i \mathbf{x}\big) < 0. \qquad (11)$$

Also, from (8), $\dot{V}(\mathbf{x}) = 2 \sum_{i=1}^{N} \dot{\mathbf{x}}_i(t)^\top P_i \mathbf{x}_i(t)$.

Note that since each nonlinearity $\boldsymbol{\phi}_i(\mathbf{x})$ verifies a sector condition as in (5), inequality (11) defines an upper bound for the time derivative of the Lyapunov function (8), implying $\dot{V}(\mathbf{x}) < 0$ $\forall \mathbf{x} \neq 0$. This completes the proof. ■

Remark 5: Due to the multiple summations of continuous membership functions in (9), there would be an uncountable number of inequalities to be checked in Theorem 1, despite the finite number of matrices used in the system description. There are different ways to relax the constraints with multiple summations to generate a finite number of LMI constraints (see Lemma 2 in the Appendix). However, when the number of subsystems N is large, these methods become impractical since a large number of numerically intractable constraints are generated. This happens due to a large number of combinations between the subsystems' membership functions. To overcome this problem, in [28], we incorporated the membership functions into the state vector so that only one LMI constraint was generated. Unfortunately, this approach has the drawback that the higher the number of subsystems the greater the order, which makes the solution nonscalable for LSSs.

Remark 6: As in [28, Remark 1], three different structures of control can be obtained trough modifications in (6), as follows: 1) decentralized control ($\Gamma_i = \mathbf{0}_{n_{u_i} \times d_{ii}}$ and $F_{ij} = \mathbf{0}_{n_{u_i} \times n_{x_j}}$); 2) linear ($\Gamma_i = \mathbf{0}_{n_{u_i} \times d_{ii}}$); and 3) nonlinear ($F_{ij} = \mathbf{0}_{n_{u_i} \times n_{x_j}}$) distributed control.

In this article, we use the chordal decomposition theorem to divide constraint (9) into smaller constraints that take into account only the subsystems that influence each other in the network. Such information is obtained from the set of maximal cliques by which the graph associated with the LSS is divided. After that, the fuzzy relaxation in Lemma 2 can be used to generate a finite number of LMI constraints, avoiding the issues discussed in Remark 5.

Constraint (9) in Theorem 1 presents block matrices that depend not only on the state but also on the nonlinearity. Hence, auxiliary matrices for decomposing the overlapping elements related to nonlinearities are required. Based on this, we define the vector $\gamma_i = (Z[\mathcal{N}_i, i])^{\circ(-1)} \in \mathbb{R}^{d_{ii}}$, which is composed by the inverses of the elements in the ith column of matrix Z defined in (1), and contained in the neighborhood $\mathcal{N}_i$ of the ith subsystem. Thus, the matrices of averaging factors for decomposing the overlapping elements related to nonlinearities are defined as follows:

$$\boldsymbol{W} = \big[W_{ij}\big], \quad \text{with } W_{ij} = \gamma_i \otimes \mathbf{1}_{n_j}^\top$$
$$\boldsymbol{Y} = \big[Y_{ij}\big], \quad \text{with } Y_{ij} = \begin{cases} \text{diag}(\gamma_i), & \text{if } i = j \\ \mathbf{0}_{d_{ii} \times d_{jj}}, & \text{otherwise.} \end{cases}$$

From Lemma 1 (block-chordal decomposition lemma), the next theorem provides sufficient conditions to ensure that the origin of the large-scale nonlinear system is asymptotically stable.

Theorem 2: Let the matrices $\boldsymbol{\Omega}_i$ in (5) be given $\forall i \in \mathcal{I}_N$. The continuous-time large-scale nonlinear system represented by the undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, with a chordal extension that has maximal cliques $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$, and composed by N interconnected closed-loop subsystems given in (7), is stable if there exist matrices $Q_i \succ 0$, diagonal matrices $\Lambda_i \succ 0$, and any matrices $R_i^{\iota_{i,2}}$, S_{ij}, and T_i $\forall i \in \mathcal{I}_N$ $\forall \iota_{i,2} \in \mathcal{I}_{r_i}$, and $\forall j \in \mathcal{N}_i$, satisfying (12) $\forall k \in \mathcal{I}_t$

$$\prod_{i \in \mathcal{C}_k} \left(\sum_{\iota_{i,1}=1}^{r_i} \sum_{\iota_{i,2}=1}^{r_i} \varsigma_i^{\iota_{i,1}} \varsigma_i^{\iota_{i,2}} \right) \begin{bmatrix} \boldsymbol{\Theta}^k & \star \\ \boldsymbol{\Pi}^k & -2\boldsymbol{\Lambda}^k \end{bmatrix} \prec 0 \qquad (12)$$

where

$$\boldsymbol{\Theta}^k = \boldsymbol{E}_{\mathcal{C}_k} (\boldsymbol{V} \circ \boldsymbol{\Theta}) \boldsymbol{E}_{\mathcal{C}_k}^\top$$
$$\boldsymbol{\Pi}^k = E_{\mathcal{E}_k} (\boldsymbol{W} \circ \boldsymbol{\Pi}) \boldsymbol{E}_{\mathcal{C}_k}^\top$$
$$\boldsymbol{\Lambda}^k = E_{\mathcal{E}_k} (\boldsymbol{Y} \circ \boldsymbol{\Lambda}_N) E_{\mathcal{E}_k}^\top$$

with $\boldsymbol{\Theta}$, $\boldsymbol{\Pi}$, and $\boldsymbol{\Lambda}_N$ given in Theorem 1, $\boldsymbol{E}_{\mathcal{C}_k} = \boldsymbol{E}[\mathcal{C}_k, \mathcal{V}]$ with $\boldsymbol{E} = \text{diag}(I_{n_{x_1}}, \ldots, I_{n_{x_N}})$, and

$$E_{\mathcal{E}_k} \in \mathbb{R}^{|\mathcal{E}_k| \times n_\phi}, \;\; \big(E_{\mathcal{E}_k}\big)_{ij} = \begin{cases} 1, & \text{if } \mathcal{E}_k(i) = \mathcal{E}(j) \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{E}_k(i)$ and $\mathcal{E}(j)$ are the ith and jth edges in $\mathcal{G}_{\mathcal{C}_k}$ and $\mathcal{G}$, respectively, sorted in the natural ordering.

Then, the control gains in (6) are recovered from:

$$K_i^{\iota_{i,1}} = R_i^{\iota_{i,1}} Q_i^{-1}, \;\; \Gamma_{ij} = T_{ij} \Lambda_{ij}^{-1} \text{ and } F_{ij} = S_{ij} Q_j^{-1}.$$

Proof: From Lemma 1, premultiplying and postmultiplying (12) by $\text{diag}(\boldsymbol{E}_{\mathcal{C}_k}^\top, E_{\mathcal{E}_k}^\top)$ and its transpose, and applying the summation on $k \in \mathcal{I}_t$, shows that constraints (12) are equivalent to constraint (9) in Theorem 1. The rest of the proof is a direct consequence of the proof of Theorem 1. ■

Remark 7: Note that if the number of maximal cliques in the graph is equal to one and it actually represents interconnections in the large-scale system, that is, the graph is complete, then Theorem 2 is reduced to Theorem 1. Furthermore, the chordal extension of the graph is used only to obtain the set of maximal cliques and indicates what subsystems participate in each constraint, while the original graph is taken into account to assemble the block matrices for each constraint. Thus, this procedure becomes a solution to Problem 1.

Remark 8: The constraints (12) in Theorem 2 are nonlinear ones, such that the use of Lemma 2 (see the Appendix) is necessary to obtain a finite number of LMI constraints. Interestingly, the smaller the number of vertices in maximal cliques, the smaller the number of combinations among membership functions. Hence the complexity is reduced with respect to Theorem 1.

Remark 9: The main advantage of Theorem 2 over Theorem 1 is the possibility of dealing with a large number of interconnected subsystems by taking into account the sparsity of the graph that represents the LSS. The higher the sparsity of the graph, the smaller the size of the LMI constraints to be considered. A graph with high sparsity means that it has a large number of maximal cliques with a small number of vertices in each clique.

V. Plug-in-Play Operation

In the previous section, we derived sufficient conditions to design distributed controllers from properties of chordal graphs, which will be used as starting point to derive proper PnP operations for interconnected subsystems. In this section we present a framework for updating the distributed controllers when the subsystems are added or removed from the LSS. The goal is to preserve the asymptotic stability for the new closed-loop LSS.

The developments in Sections V-A and V-B show that the proposed approach is independent of any underlying FDI system that could, in principle, be designed separately, while providing a definite solution to Problem 2.

A. Plugging-in Operation

Consider the plug-in of a new nonlinear subsystem S_{N+1} described by (3) with matrices $A^l_{N+1,N+1}$, $A^l_{N+1,j}$, B^l_{N+1}, G^l_{N+1} $\forall j \in \mathcal{N}_{N+1}$. In particular, $\mathcal{N}_{N+1}$ identifies the subsystems that are directly coupled to S_{N+1}, that is, its neighborhood.

Fig. 2 depicts the plugging-in operation of the 6th subsystem (S_6) to an LSS composed by five subsystems. The graph associated to the new LSS generated after adding subsystem S_6 is denoted by $\mathcal{G}^+(\mathcal{V}^+, \mathcal{E}^+)$, where $\mathcal{V}^+ = \mathcal{V} \cup \{N+1\}$ and $\mathcal{E}^+ = \mathcal{E} \cup \{(N+1, j)_{j \in \mathcal{N}_{N+1}}\}$. Note that subsystems S_j $\forall j \in \mathcal{N}_{N+1}$, have the new neighbor S_{N+1}.

The goal of the plugging-in operation is to guarantee closed-loop stability of LSS without requiring to redesign all subsystems' controllers. According to the demonstration in Section IV, each clique of the chordal extension of the graph represents a part of the LSS and is related to others through overlapping elements, that is, subsystems that belong to more than one clique. Thus, the overall stability of the LSS is guaranteed from the local stability of cliques. After that, if a new subsystem is added to the LSS, it is still necessary to ensure the cliques' stability to which that subsystem and its neighborhood belong, that is, the stability of those cliques that contain the subsystems in the set $\mathcal{N}^+_{N+1} = \{N+1\} \cup \mathcal{N}_{N+1}$.

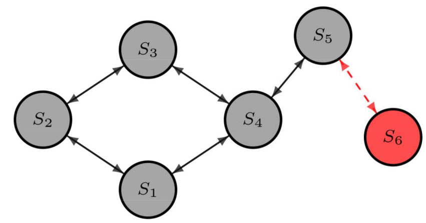

Fig. 2. Plugging-in operation of the 6th subsystem (S_6). Dashed arrow (red) is the new connection generated by adding the new subsystem.

Remark 10: Note that since $F_{ij} = S_{ij} Q_j^{-1}$ in (6), the neighborhood of the newly added $(N+1)$th subsystem will have their controllers and respective block in the Lyapunov matrix updated. If (6) was used, it would require updating the blocks of the Lyapunov matrix related to the subsystems connected to the new subsystem. If this is necessary, constraints regarding the cliques that contain these subsystems should be analyzed again. This would occur in a chain and all constraints would have to be eventually analyzed only to find the new blocks of the Lyapunov matrix, and this would hinder one of the main advantages of the PnP approach. To overcome this problem, we consider specifically in the PnP approach that $F_{ij} = \mathbf{0}_{n_{u_i} \times n_{x_j}}$ in (6), such that the distributed control law for every ith subsystem in the LSS is simplified in this case to

$$\mathbf{u}_i(t) = \sum_{l=1}^{r_i} \varsigma_i^l(\mathbf{z}_i(t)) K_i^l \mathbf{x}_i(t) + \Gamma_i \boldsymbol{\phi}_i(\mathbf{x}(t)). \tag{13}$$

The next theorem provides sufficient conditions to ensure that the origin of an asymptotic stable large-scale nonlinear system, whose subsystems are controlled using (13), remains asymptotically stable when it is subject to the plugging-in operation of a new subsystem as described above.

Theorem 3 (Plugging-In): Let the matrices $\boldsymbol{\Omega}_i$ in (5) be given $\forall i \in \mathcal{N}^+_{N+1} = \{N+1\} \cup \mathcal{N}_{N+1}$. The plugging-in operation of a nonlinear subsystem $N+1$, controlled using (13), to a previously asymptotically stable continuous-time large-scale nonlinear system, whose subsystems are also controlled using (13) with gains designed using Theorem 2 (with $S_{ij} = 0$), leading to the new undirected graph $\mathcal{G}^+(\mathcal{V}^+, \mathcal{E}^+)$ with a corresponding chordal extension that has maximal cliques $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$, remains asymptotically stable if there exist matrices $Q_i \succ 0$, diagonal matrices $\Lambda_i \succ 0$, and any matrices $R_i^{\iota_{i,2}}$ and T_i, $i \in \mathcal{N}^+_{N+1}$, and $\forall \iota_{i,2} \in \mathcal{I}_{r_i}$, satisfying (14) $\forall k$ such that $\mathcal{N}^+_{N+1} \cap \mathcal{C}_k \neq \emptyset$

$$\prod_{s \in \mathcal{C}_k} \left(\sum_{\iota_{s,1}=1}^{r_s} \sum_{\iota_{s,2}=1}^{r_s} \varsigma_s^{\iota_{s,1}} \varsigma_s^{\iota_{s,2}} \right) \begin{bmatrix} \boldsymbol{\Theta}^k & \star \\ \boldsymbol{\Pi}^k & -2\boldsymbol{\Lambda}^k \end{bmatrix} \prec 0 \tag{14}$$

where $\mathbf{\Theta}^k$, $\mathbf{\Pi}^k$, and $\mathbf{\Lambda}^k$ are given as in Theorem 2 (replacing $S_{ij} = 0$ in all expressions), while considering $\mathcal{G}^+(\mathcal{V}^+, \mathcal{E}^+)$ instead of $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

Then, the control gains in (13) for the added subsystem and the subsystems in its neighborhood are recovered from:

$$K_i^{\iota_{i,1}} = R_i^{\iota_{i,1}} Q_i^{-1} \text{ and } \Gamma_{ij} = T_{ij}\Lambda_{ij}^{-1}.$$

Proof: Since the closed-loop LSS prior to the plugging-in operation is asymptotically stable and its subsystems' controllers were designed from Theorem 2 with $S_{ij} = 0 \Rightarrow F_{ij} = \mathbf{0}_{n_{u_i} \times n_{x_j}}$, LMI constraint (14) takes only into account cliques such that $\mathcal{N}_{N+1}^+ \cap \mathcal{C}_k \neq \emptyset$, and from Theorem 2, the already known controllers of other subsystems are solutions to cliques $\mathcal{C}_k$ such that $\mathcal{N}_{N+1}^+ \cap \mathcal{C}_k = \emptyset$. Thus, the solution for (14) together with the previously known controllers are also solutions to Theorem 2, when Remark 10 and the LSS with the associated graph $\mathcal{G}^+(\mathcal{V}^+, \mathcal{E}^+)$ are taken into consideration. ■

Remark 11: Note that in Theorem 3, if $s \notin \mathcal{N}_{N+1}^+$ in (14), the controller of the referred subsystem should not be modified, and the controller already known is used to build the respective LMI constraint.

Remark 12: The redesign of controllers in the neighborhood of the $(N+1)$th subsystem is necessary because such subsystems can become part of a new clique. Thus, the matrix blocks corresponding to them are overlapping elements that must be divided between the constraints formed by the cliques (see Section IV).

B. Unplugging Operation

The unplugging operation consists in removing a subsystem from the network. This scenario is mainly due to a failure in a subsystem. Consider the unplugging of a nonlinear subsystem S_ρ with $\rho \in \mathcal{I}_N$, described by (3). The graph associated to new LSS generated after removing the ρth subsystem is denoted by $\mathcal{G}^-(\mathcal{V}^-, \mathcal{E}^-)$, where $\mathcal{V}^- = \mathcal{V} - \{\rho\}$ and $\mathcal{E}^- = \mathcal{E} - \{(\rho, j)_{j \in \mathcal{N}_\rho}\} + \{(j, \rho)_{j \in \mathcal{N}_\rho}\}$.

Similar to the plugging-in case, the goal of unplugging operation is to guarantee that the LSS will remain asymptotically stable without requiring the redesigning of all subsystems' controllers. However, there are a few differences between these operations. For example, removing a system from the LSS can lead to the creation of various smaller LSSs. This happens when the resulting undirected graph associated with the new LSS is not connected. Interestingly, the result below is generic enough to also be utilized in this situation.

As discussed previously, the overall stability of the LSS is guaranteed from the local stability of its cliques. Therefore, if a subsystem is removed from the LSS, it is only necessary to ensure the cliques' stability to which the subsystems in its former neighborhood $\mathcal{N}_\rho$ belong. In addition, only the local gains $K_i^{\iota_{i,1}}$ $\forall i \in \mathcal{N}_\rho$, should be updated since removing a subsystem changes the degrees of the vertices that belong to the set $\mathcal{N}_\rho$ and the resulting overlap of the cliques.

The next theorem provides sufficient conditions to ensure that the origin of the large-scale nonlinear system is asymptotically stable when it is subject to unplugging of a subsystem.

Theorem 4 (Unplugging): Let the matrices $\mathbf{\Omega}_i$ in (5) be given $\forall i \in \mathcal{N}_\rho$. The unplugging operation of a nonlinear subsystem ρ, controlled using (13), from a previously asymptotically stable continuous-time large-scale nonlinear system, whose subsystems are also controlled using (13) with gains designed using Theorem 2 (with $S_{ij} = 0$), leading to the new graph $\mathcal{G}^-(\mathcal{V}^-, \mathcal{E}^-)$ with a chordal extension that has maximal cliques $\{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_t\}$, remains asymptotically stable if there exist matrices $Q_i \succ 0$, and any matrices $R_i^{\iota_{i,2}}$ $\forall i \in \mathcal{N}_\rho$, and $\forall \iota_{i,2} \in \mathcal{I}_{r_i}$, satisfying (15) $\forall k$ such that $\mathcal{N}_\rho \cap \mathcal{C}_k \neq \emptyset$

$$\prod_{s \in \mathcal{C}_k} \left(\sum_{\iota_{s,1}=1}^{r_s} \sum_{\iota_{s,2}=1}^{r_s} \varsigma_s^{\iota_{s,1}} \varsigma_s^{\iota_{s,2}} \right) \begin{bmatrix} \mathbf{\Theta}^k & \star \\ \mathbf{\Pi}^k & -2\mathbf{\Lambda}^k \end{bmatrix} \prec 0 \quad (15)$$

where $\mathbf{\Theta}^k$, $\mathbf{\Pi}^k$, and $\mathbf{\Lambda}^k$ are given as in Theorem 2 (replacing $S_{ij} = 0$ in all expressions), while considering $\mathcal{G}^-(\mathcal{V}^-, \mathcal{E}^-)$ instead of $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

Then, the local control gains in (13) for the subsystems in the neighborhood of the removed ρth subsystem are recovered from

$$K_i^{\iota_{i,1}} = R_i^{\iota_{i,1}} Q_i^{-1}.$$

Proof: Since the closed-loop LSS prior to the unplugging operation is asymptotically stable and its subsystems' controllers were designed from Theorem 2 with $S_{ij} = 0 \Rightarrow F_{ij} = \mathbf{0}_{n_{u_i} \times n_{x_j}}$, LMI constraint (15) takes only into account cliques such that $\mathcal{N}_\rho \cap \mathcal{C}_k \neq \emptyset$, and from Theorem 2, the already known controllers of others subsystems are solutions to cliques such that $\mathcal{N}_\rho \cap \mathcal{C}_k = \emptyset$. Thus, the solution for (15) together with the already known controllers are also solutions to Theorem 2, when Remark 10 and the LSS with associated graph $\mathcal{G}^-(\mathcal{V}^-, \mathcal{E}^-)$ are properly considered. ■

Remark 13: Similar to the plugging-in case, if $s \notin \mathcal{N}_\rho$ in (15), the controller already known for the sth subsystem is used to build the respective LMI constraint.

Remark 14: Both plugging-in and unplugging operations can be used in the design of mixed distributed and decentralized controllers for the LSS. Indeed, decentralized controllers will be obtained for the subsystems that should be updated after these operations by just keeping the corresponding matrices $T_i = 0$ in all the expressions in Theorem 3 (plugging-in) and in Theorem 4 (unplugging), while assuming that feasible solutions will be found.

VI. Coupled Van der Pol Oscillators

In this section, we use the coupled Van der Pol oscillators to illustrate the effectiveness of the proposed methodologies. Such oscillators have been used to represent oscillatory systems in many diverse areas, from electronic oscillators to biological rhythms, since they capture the general concept of relaxation oscillators [49].

Inspired by [38], where linear interconnections are considered, a network of Van der Pol oscillators composing a continuous-time LSS is considered in this section, where nonlinear interconnections instead of linear ones couple them.

Each ith subsystem is described as follows:

$$\begin{aligned}\dot{x}_{i1}(t) &= x_{i2}(t)\\ \dot{x}_{i2}(t) &= -x_{i1}(t) + \mu_i\Big(1 - x_{i1}^2(t)\Big)x_{i2}(t) + g_i(x_{i1}(t))u_i(t)\\ &\quad + \gamma_i \sum_{j\in\mathcal{N}_i} \sin\big(x_{j1}(t) - x_{i1}(t)\big)\end{aligned}$$

where $i \in \mathcal{I}_N$, N is the number of oscillators, $g_i(x_{i1}(t)) = (1/[0.4 + 0.1x_{i1}^2(t)])$ is the function describing the nonlinear dynamics of ith actuator, the state vector of the ith subsystem is $\mathbf{x}_i(t) = \begin{bmatrix} x_{i1}(t) & x_{i2}(t)\end{bmatrix}^\top$, $x_{i1}(t)$ is the displacement from the equilibrium position, $x_{i2}(t)$ is the corresponding velocity, and $u_i(t)$ is the force applied to the ith oscillator. We chose the parameters μ_i and γ_i randomly inside the intervals $[0.5,\ 1]$ and $[-0.5,\ 0.5]$, respectively.

An exact 4-rule N-TS fuzzy model (3) can be obtained to each ith oscillator applying the sector nonlinearity approach when $|x_{i1}(t)| \le 3$. $z_{i1}(t) = x_{i1}(t)$ and $z_{i2}(t) = g_i(x_{i1}(t))$ were chosen as the premise variables of the ith subsystem and $\varphi_{ij_\kappa}(\mathbf{x}_i,\ \mathbf{x}_{j_\kappa}) = (x_{i1} - x_{j_\kappa 1}) + \sin(x_{j_\kappa 1} - x_{i1}) \in \mathrm{co}\{0,\ \mathbf{\Omega}_{i(\kappa)}\mathbf{x}\}$, with $\kappa \in \mathcal{I}_{d_{ii}}$, and $\mathbf{\Omega}_i = \begin{bmatrix}\Omega_{i1} & \Omega_{i2} & \cdots & \Omega_{iN}\end{bmatrix}$, $\Omega_{ii(\kappa)} = \begin{bmatrix}1.22 & 0\end{bmatrix}$ $\forall i \in \mathcal{I}_N$, for $\Omega_{ij_\kappa(\kappa)} = -\Omega_{ii(\kappa)}$, if $j_\kappa \in \mathcal{N}_i$ and $\Omega_{ij(\kappa)} = \begin{bmatrix}0 & 0\end{bmatrix}$, otherwise. The membership functions are $\varsigma_i^1(\mathbf{z}_i) = w_0^1(z_{i1})w_0^2(z_{i2})$, $\varsigma_i^2(\mathbf{z}_i) = w_0^1(z_{i1})w_1^2(z_{i2})$, $\varsigma_i^3(\mathbf{z}_i) = w_1^1(z_{i1})w_0^2(z_{i2})$, and $\varsigma_i^4(\mathbf{z}_i) = w_1^1(z_{i1})w_1^2(z_{i2})$, where

$$w_0^1(z_{i1}) = \frac{z_{i1}^2}{9}, \quad w_0^2(z_{i2}) = \frac{z_{i2} - 0.7692}{1.7308}$$

and $w_1^k(z_{ik}) = 1 - w_0^k(z_{ik})$, $k \in \mathcal{I}_2$. Thus, the ith local state-space matrices are given by

$$\begin{aligned} A_i^1 &= A_i^2 = \begin{bmatrix} 0 & 1\\ -1 - d_{ii}\gamma_i & -8\mu_i\end{bmatrix}\\ A_i^3 &= A_i^4 = \begin{bmatrix} 0 & 1\\ -1 - d_{ii}\gamma_i & \mu_i\end{bmatrix}\\ B_i^1 &= B_i^3 = \begin{bmatrix} 0\\ 2.5\end{bmatrix}, \quad B_i^2 = B_i^4 = \begin{bmatrix} 0\\ 0.7692\end{bmatrix}\\ A_{ij}^1 &= A_{ij}^2 = A_{ij}^3 = A_{ij}^4 = \begin{bmatrix} 0 & 0\\ \gamma_i & 0\end{bmatrix}\\ G_i^1 &= G_i^2 = G_i^3 = G_i^4 = \begin{bmatrix} 0\\ \gamma_i\end{bmatrix}\end{aligned}$$

where $d_{ii} = |\mathcal{N}_i|$ is the degree of the ith subsystem.

Initially, Theorem 2 is used to ensure the stability of LSS composed by $N = 50$ subsystems in which their interconnections are represented by the graph in Fig. 3, not considering subsystems #51, #52 and #53 (blue squares). Figs. 4 and 5 depict the closed-loop behavior of the LSS, with initial conditions $x_{i10} \in [-3,\ 3]$ and $x_{i20} = 0$ $\forall i \in \mathcal{I}_N$. Both figures present an inset picture detailing what happens for $t \in [0,\ 3]$ to highlight the stabilization of the subsystems.

The subsystems #28 and #29 (red triangles) will be unplugged from the LSS. However, note that the unplugging operation of the subsystem #29 would create four smaller LSSs in the absence of subsystems #51, #52, and #53. Therefore, these subsystems are sequentially plugged in the LSS (Theorem 3), within separation time intervals of 5 s for each operation, before the unplugging operations (Theorem 4) of the subsystems #28 and #29, which also occur sequentially at every 5 s. Since the controllers of the now isolated #28 and #29 subsystems are isolated after the unplugging operation, they start to oscillate. Note that the proposed distributed controller guarantees the asymptotic stability of the closed-loop

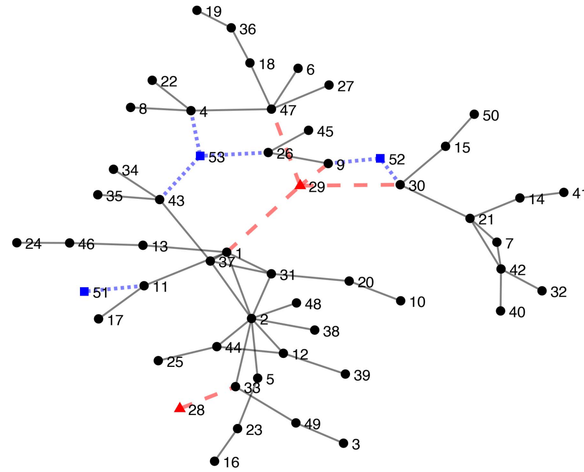

Fig. 3. Graph of the LSS. The unplugging operation is made in subsystems #28 and #29 (red triangles). The plugging operation is made in subsystems #51, #52, and #53 (blue squares).

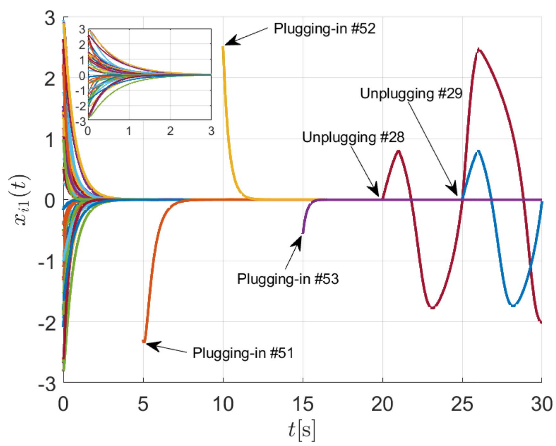

Fig. 4. Oscillators' displacements trajectories ($x_{i1}(t)$) from the equilibrium position.

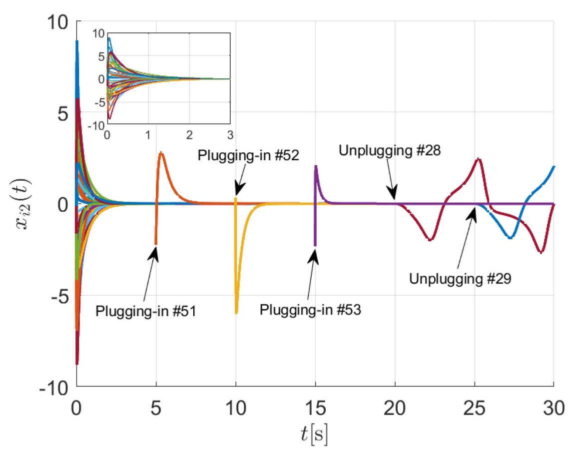

Fig. 5. Oscillators' velocities trajectories ($x_{i2}(t)$).

LSS and the PnP approach is effective when subsystems are added or removed from the network.

VII. Conclusion

In this work a PnP distributed control approach for stabilizing continuous-time large-scale nonlinear systems was proposed. From the combination of the chordal decomposition theory and the multiple fuzzy summations technique, it was possible to extend the approach of using fuzzy N-TS models proposed in [28] for the large-scale case.

Plugging and unplugging operations were proposed to derive a complete PnP methodology for LSSs. Both operations can ensure the overall system's stability through only the *local* reconfiguration of controllers obtained from convex design conditions in terms of LMIs derived based on a quadratic block-diagonal Lyapunov function. Besides that, the proposed approach is flexible since it can be used to obtain mixed distributed and decentralized controllers and allows the joint use of any off-the-shelf FDI system. A network of coupled Van der Pol oscillators has been used to illustrate the effectiveness of the proposed PnP approach.

We note that both the location of the subsystems in the graph and the strength of the interactions impact the control performance. We plan to investigate these issues in-depth in future research. That also includes using the nonquadratic Lyapunov function candidates and considering time delay in the control laws as well as in the interconnections among subsystems.

Appendix
Multiple Fuzzy Summations

The analysis and synthesis conditions of N interconnected N-TS fuzzy systems usually present N sets of double fuzzy summations, that is, each subsystem has corresponding double fuzzy summations. In other words, each subsystem has its proper set of membership functions. Based on this, fuzzy relaxations commonly used in TS fuzzy systems [25], [27] are not adequate to address this scenario, because they consider only one set of fuzzy summations and a generalization to N sets is required. Hence, we use the notion of multisets [50] that emerges due to multiple sets of membership functions. A similar idea has been utilized in the context of discrete-time systems [47], [51], [52] concerning multisets of delays, which are inserted in the membership function to reduce the conservativeness.

Next, we review fundamental concepts for multisets. The notation used is based on the one proposed in [47].

Definition 3 (Multisets [50]): Let $\mathcal{S} = \{\varsigma_1, \varsigma_2, \ldots, \varsigma_n\}$ be a set. A multiset $\mathcal{S}_\varsigma$ over $\mathcal{S}$ is a cardinal-valued function $\mathcal{S}_\varsigma : \mathcal{S} \mapsto \mathbb{N}$ such that for $\varsigma \in \mathrm{Dom}(\mathcal{S}_\varsigma)$ implies the cardinal $|\varsigma|_{\mathcal{S}_\varsigma}$. The value $|\varsigma|_{\mathcal{S}_\varsigma}$ denotes the multiplicity of ς, that is, the number of times ς occurs in $\mathcal{S}_\varsigma$. A multiset $\mathcal{S}_\varsigma$ is denoted here by the set of pairs $\mathcal{S}_\varsigma = \{\langle |\varsigma_1|_{\mathcal{S}_\varsigma}, \varsigma_1\rangle, \ldots, \langle |\varsigma_n|_{\mathcal{S}_\varsigma}, \varsigma_n\rangle\}$.

Remark 15: If the multiplicity of a given element $\varsigma \in \mathcal{S}_\varsigma$ is 1, it is simply denoted $\langle 1, \varsigma\rangle = \varsigma$. Particularly, synthesis conditions in this article present only multisets of double fuzzy summations, that is, the multiplicity of membership functions for the ith subsystem is $|\varsigma_i|_{\mathcal{S}_\varsigma} = 2$ and the multiset of membership functions associated to the overall system is $\mathcal{S}_\varsigma = \{\langle 2, \varsigma_1\rangle, \ldots, \langle 2, \varsigma_n\rangle\}$.

Definition 4 (Index Set and Multi-Index): The ith index set of a multiple fuzzy summation with the multiset of membership functions $\mathcal{S}_\varsigma$ is the set of all indexes in the sum associated with subset $\mathcal{S}_{\varsigma_i} \subset \mathcal{S}_\varsigma$. It is denoted as

$$\mathbb{I}_{\varsigma_i} = \left\{ \boldsymbol{\iota}_i = \left(\iota_{i,1}, \ldots, \iota_{i,|\mathcal{S}_{\varsigma_i}|}\right) : \iota_{i,l} \in \mathcal{I}_{r_i},\ l \in \mathcal{I}_{|\mathcal{S}_{\varsigma_i}|} \right\}.$$

An element $\boldsymbol{\iota}_i \in \mathbb{I}_{\varsigma_i}$ is called the multi-index of ith subset of multiset $\mathcal{S}_\varsigma$.

Lemma 2 (Multiple Fuzzy Summation With Multiset of Membership Functions, Adapted From [47, Lemma 2]): A sufficient condition for the satisfaction of the following inequality dependent on a multiple fuzzy summation with the multiset of membership functions:

$$\sum_{\boldsymbol{\iota}_1 \in \mathbb{I}_{\varsigma_1}} \cdots \sum_{\boldsymbol{\iota}_N \in \mathbb{I}_{\varsigma_N}} \left(\prod_{i=1}^{N} \varsigma_i^{\boldsymbol{\iota}_i} \right) \Upsilon_{(\boldsymbol{\iota}_1, \ldots, \boldsymbol{\iota}_N)} \succ 0$$

is that for every combination of $(\boldsymbol{\iota}_1, \ldots, \boldsymbol{\iota}_N)$, where $\boldsymbol{\iota}_i$ is a multi-index given by Definition 4, the sum of its permutations is positive definite.

Lemma 2 is obtained from the recursive application of Polya's theorem [53] in each ith subset of multiset $\mathcal{S}_\varsigma$ associated with multiple fuzzy summation.

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